Patterns in Calculations Involving Molar Solubility and K_{sp}

1. For a simple binary ionic compound with low solubility, the chemical equation for dissolution is:

   $CA_{(s)} \text{->} C^{+}_{(aq)} + A^{-}_{(aq)}$

   and the solubility product (equilibrium constant) for the reaction is:

   $K_{sp} = [C^{+}]\ [\ A^{-}]$

   where $[C^{+}]$ is the molar concentration of the cation and $[\ A^{-}]$ is the molar concentration of the anion.

   If the only species in solution are the products of CA, and its molar solubility is x, then

   $K_{sp} = x^{2}$

2. If the compound has the formula C_2A:

   $C_2A_{(s)} \text{->} 2C^{+}_{(aq)} + A^{2-}_{(aq)}$

   and

   $K_{sp} = [2C^{+}]^{2}\ [A^{2-}]$

   If the only species in solution are the products of C_2A, and its molar solubility is x, then

   $K_{sp} = 4x^{3}$

   For CA_2

   $CA_{2(s)} \text{->} C^{2+}_{(aq)} + 2A^{-}_{(aq)}$

   and

   $K_{sp} = [C^{2+}]\ [2A^{-}]^{2}$

   Like C_2A, CA_2 with a molar solubility of x is

   $K_{sp} = 4x^{3}$

3. C_2A_2 is less common, but results in:

   $K_{sp} = 16x^{4}$

4. C_3A and CA_3 result in:

   $K_{sp} = 27x^{4}$

5. C_3A_2 and C_2A_3 result in:

   $K_{sp} = 108x^{5}$

Although other ion ratios are possible, they are rare. Follow the pattern demonstrated above to solve those questions.

Summarizing, to find K_{sp} when molar solubility is known:

Formula	K_{sp}
CA	$K_{sp} = x^2$
C_2A or CA_2	$K_{sp} = 4x^3$
C_2A_2	$K_{sp} = 16x^4$
C_3A or CA_3	$K_{sp} = 27x^4$
C_3A_2 or C_2A_3	$K_{sp} = 108x^5$

To find the molar solubility when K_{sp} is known:

Formula	Molar Solubility, x
CA	$x = \sqrt{K_{sp}}$
C_2A or CA_2	$x = \sqrt[3]{(K_{sp}/4)}$
C_2A_2	$x = \sqrt[4]{(K_{sp}/16)}$
C_3A or CA_3	$x = \sqrt[4]{(K_{sp}/27)}$
C_3A_2 or C_2A_3	$x = \sqrt[5]{(K_{sp}/108)}$

When two or more compounds contribute to the ions in solution, the common ion effect determines whether precipitation will take place. To determine whether a particular concentration of ion pairs will create a precipitate, and K_{sp} is known:

Formula	Solubility Quotient, Q_{sp}
CA	$Q_{sp} = [C^+]\ [A^-]$
C_2A	$Q_{sp} = [C^+]^2\ [A^{2-}]$
CA_2	$Q_{sp} = [C^{2+}]\ [A^-]^2$
C_2A_2	$Q_{sp} = [C^+]^2\ [A^-]^2$
C_3A	$Q_{sp} = [C^+]^3\ [A^{3-}]$
CA_3	$Q_{sp} = [C^{3+}]\ [A^-]^3$
C_3A_2	$Q_{sp} = [C^{2+}]^3\ [A^{3-}]^2$
C_2A_3	$Q_{sp} = [C^{3+}]^2\ [A^{2-}]^3$

Calculate the Q_{sp} based on the given concentrations and the formula.

If $Q_{sp} > K_{sp}$ then a precipitate will form.

If $Q_{sp} < K_{sp}$ then a precipitate will not form.

If $Q_{sp} = K_{sp}$ the system is at equilibrium.